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## Independent - Dependent Demand



Spares

## Why inventories?



- Economies of Scale
- Supply and Demand Uncertainty
- Volume Discounts/Impending Price Rise
- Long Lead Times and Quick Response to Customer's Demand
- To maintain independence of operations
- To allow flexibility in production scheduling


## Inventory



## Inventory is injurious to your health!



Get Lean...Get healthy!


# We want to turn our inventory faster than our people <br> - A quote by James D. Sinegal <br> - Co-founder, Costco 



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INVENTORY TURNS SHOULD ALWAYS BE


## Inventory classification

- Classification by form
- Raw Materials (RM)
- Work-in-Process (WIP)
- Finished Goods (FG)
- Classification by Life cycle
- Perishable
- Non-perishable


## Inventory classification by function

- Cyclic stock
- Ordering lot size/2
- Safety stock

- To protect against uncertainties
- Pipeline
- Scheduled receipts or open orders

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## Cyclic, Pipeline and Safety Stocks



Cyclic inventory, pipeline inventory and safety stocks are critically linked to "how much" and "when" decisions in inventory planning

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## Costs of Inventory

- Physical holding cost (out-of-pocket)
- Financial holding cost (opportunity cost)
- Holding (or carrying) costs
- Production change
- Product changeover
- Setup costs
- Low responsiveness
- to demand/market changes
- Shortage costs
- to supply/quality changes
- Managerial and clerical costs
- Ordering Costs
- Transportation costs


## Inventory Policy parameters

- WHEN to order?
- HOW MUCH to order?
- In WHAT FORM? (RM, WIP or FG)
- WHERE TO DEPLOY in the supply chain?


## Types of inventory models



## Single Period Deterministic

- You have to make a decision on how much to inventory in every period
- You know how much the demand for the period is going to be
-What do you do?


## Multi Period Deterministic

- Perpetual inventory system
- Demand for the product is known constant and uniform throughout the period
- Lead time (time from ordering to receipt) is constant
- Replenishment is instantaneous
- Price per unit of product is constant
- Inventory holding cost is based on average inventory
- Ordering or setup costs are constant
- All demands for the product will be satisfied (no back orders are allowed)
- How would the inventory level look like?

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## EOQ model

$D$ : Demand per year
$S$ : Setup or Order Cost
(Rs/Setup; Rs/Order)
$c=$ unit cost of item
$h=$ Inv holding cost rate
$H=h c$ : Inventory holding cost
(Rs./year/unit)
$Q$ : Order quantity

$T$ : Reorder cycle

$$
T C\left(Q^{*}\right)=\sqrt{2 S D H}
$$

## Inventory Planning Models

| Mean of weekly demand | $: 200$ |
| :--- | :--- |
| Standard deviation of weekly demand | $: \overline{40}$ |
| Unit cost of the raw material | $: \overline{\text { Rs. } 300 /-}$ |
| Ordering cost | $:$ Rs. 460/- per order |
| Carrying cost percentage | $: 20 \%$ per annum |
| Lead time for procurement | $: 2$ weeks |
| EOQ Model | $=200$ |
| Weekly demand | $=52$ |
| Number of weeks per year | $=10,400$ |
| Annual demand, $D=200 * 52$ |  |
| Carrying cost, $C_{c}=$ Rs. 60.00 per unit per year |  |
|  |  |
| Economic Order Quantity $=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 * 460 * 10,400}{60}}=399.33 \approx 400$ |  |
| Time between orders $=\frac{400}{10400}=\frac{2}{52}=2$ weeks |  |

## Practical issues with the EOQ model

## - It may not be possible to

- Order exactly Q*
- Order as close as possible to Q*
- Estimate the parameters (D,S,H) accurately
- EOQ model is robust to small errors in these values
- Instantaneous replenishment
- Incorporate lead time using ROP level


## - Price discounts

- Use modified procedure


## Incorporating Lead time



## Certain Demand



## Price Discounts

- Why do suppliers give price discounts?
- Compute Q* values
- From lowest price to the highest
- Until valid Q* is obtained
- Compute TRC at this Q* and each price break above this Q*
- Choose the order quantity with least TC


## The elephant in the room

## Demand uncertainty!!!




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## If life were predictable it would cease to be life, and be without flavor.

Eleanor Roosevelt


## Single period Stochastic Demand

- Examples?
- Newspapers
- Cakes
- Fashion products?


## Demand characteristics

- Demand follows a normal distribution
- NORM(50,10)
- How much would you order?


## Managing the average will make you an average manager!

## A quote by



## Understanding Service level

- What area of the demand distribution would you cover?
http://homepage.divms.uiowa.edu/~mbognar/applets/normal.html


## Optimal Service level

Happiness is a mysterious thing, to be found somewhere between too little and too much


## Newsvendor model

- Inventory decision under uncertainty
- The "too much/too little problem":
- Order too much and inventory is left over at the end of the season
- Order too little and sales are lost.


## Notation

- Demand $D$ is a random variable
- Cumulative distribution function $F(D)$
- Wholesale price W
- Selling price R
- Salvage value S (<W)
- How much should the retailer order?


## "Too much" and "too little" costs

$$
W=3 \quad R=12 \quad S=1
$$

- $C_{o}=$ overage cost
- The cost of ordering one more unit than what you would have ordered had you known demand.
- Increase in profit you would have enjoyed had you ordered one unit lesser.
$-C_{o}=$ Cost - Salvage value $=W-S=2$
- $C_{u}=$ underage cost

- The cost of ordering one fewer unit than what you would have ordered had you known demand.
- Increase in profit you would have enjoyed had you ordered one unit more.
- $C_{u}=$ Price - Cost $=R-W=9$


## Balancing the risks and benefits

- Risk: Ordering one more unit increases the chance of overage
- Expected loss on the $Q^{\text {th }}$ unit $=C_{o} \times F(Q)$, where $F(Q)=$ Prob\{Demand <= Q)
- Benefit: Ordering one more unit decreases the chance of underage:
- Expected benefit on the $Q^{\text {th }}$ unit $=C_{u} \times(1-F(Q))$


## Expected profit maximizing order quantity

- To minimize the expected total cost of underage and overage, order $Q$ units so that the expected marginal cost with the $Q^{t h}$ unit equals the expected marginal benefit with the $Q^{t h}$ unit:

$$
\underline{C_{o}} \times F(Q)=C_{u} \times(1-F(Q))
$$

- Rearrange terms in the above equation ->
- The ratio $C_{u} /\left(C_{o}+C_{u}\right)$ is called the critical ratio.

$$
\begin{aligned}
& F(Q)=\frac{C_{u}}{C_{0}+C_{u}} \\
& \text { ratio. }
\end{aligned}
$$

- In other terms, (R-W)/(R-S). $\underline{R}$ and $\underline{S}$ are determined by the market.
$R=12 \quad W=3$


## What is the Optimal service level?

Let $C_{o}=$ Cost of over stocking per unit
$C_{u}=$ Cost of under stocking per unit
$Q=$ Number of units to be stocked
$d$ = Single period demand
$P(d \leq Q) \quad=$ The probability of the single period demand being at most $Q$ units

$$
P(d \leq Q) \leq \frac{C_{u}}{C_{u}+C_{o}}=\text { Service Level }
$$

## Multi period Stochastic Demand

- Extending the idea to multiple periods


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## Back to the multi-period setting



## Computing safety stock

Let the demand during lead time follow a Normal distribution Mean demand during lead-time $=\mu_{(L)}$ Standard deviation of demand during lead-time $={ }^{\sigma}{ }^{\sigma}(L)$ Desired service level $=(1-\alpha)$
The probability of a stock out $=\alpha$ Standard normal variate corresponding to an area of $(1-\alpha)$ covered on the left side of the normal curve $=Z_{\alpha}$


Safety stock (SS) is given by $\mathrm{SS}=Z_{\alpha} * \sigma_{(L)}$

## Factors effecting safety stock

- Demand uncertainty (forecast uncertainty)
- Reducing demand uncertainty (better forecast) reduces safety stock required, reducing material cycle time
- Replenishment lead time
- Reducing replenishment lead time, improves forecast accuracy, reducing safety stock required (square root factor)
- Variability of supply lead time
- Reducing variability of supply, improves forecast accuracy, reducing safety stock required


## Relationship between Service Level and Safety Stock



## Continuous Review (Q) System



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## Continuous review policy

L: Lead time
D: Average demand per unit time
$\sigma$ : Standard deviation of demand per unit time
$\sigma_{I}$ : Standard deviation of demand during lead time
ss: Safety stock
$k$ : Safety factor
$r$ : Reorder point
AI : Average inventory
AT: Material Cycle time

$$
\begin{aligned}
& \sigma_{I}=\sigma \sqrt{L} \\
& s s=k \times \sigma_{I} \\
& r=D \times L+s s \\
& A I=\frac{Q}{2}+s s \\
& A T=\frac{A I}{D}
\end{aligned}
$$

## Periodic Review (P) System



## Periodic Review policy

$L$ : Lead time
$T$ : Reorder interval
$D$ : Average demand per unit time
$\sigma$ : Standard deviation of demand per unit time
$\sigma_{I+T}$ : Standard deviation of demand during $\mathrm{L}+\mathrm{T}$ periods
$\mathrm{F}(k)$ : Cycle service level
ss: Safety stock
$k$ : Safety factor
$S$ : Order up to quantity
$A I$ : Average inventory
$A T$ : Material Cycle time
$\sigma_{I}+R=\sigma \sqrt{L+T}$
$S S=k \times \sigma_{I}+T$
$S=D \times(L+T)+S S$
$A I=\frac{D \times T}{2}+s s$
$A T=\frac{A I}{D}$

## Periodic \& Continuous Review Systems

| Criterion | Continuous Review (Q) System | Periodic Review (P) System |
| :--- | :--- | :--- |
| How much to <br> order | Fixed order qty: Q | $\mathrm{S}=\mu_{(\mathrm{L}+\mathrm{R})}+\mathrm{Z}_{\alpha} \times \sigma_{(L+T)}$ <br> $\mathrm{Q}_{R}=\mathrm{S}-\mathrm{I}_{\mathrm{T}}$ |
| When to order | $\mathrm{ROP}=\mu_{(\mathrm{L})}+\mathrm{Z}_{\alpha} \times \sigma_{(L)}$ | Every T periods |
| Safety stock | $\mathrm{SS}=\mathrm{Z}_{\mathrm{a}} \times \sigma_{(L)}$ | $\mathrm{SS}=\mathrm{Z}_{\alpha} \times \sigma_{(L+T)}$ |
| Salient aspects | - Implemented using two bin <br> system <br> - Suited for medium and low <br> value items | - More safety stock <br> - More responsive to demand <br> - Ease of implementation |

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## Hybrid Inventory Policies

- $(\mathrm{s}, \mathrm{S})$ policy
- Optional replenishment system
- Periodic review with a order upto level
- Ensures minimum ordering level
- Eliminates continuous review
- Base stock system
- Order as much as you sell
- Base stock level = expected demand during lead time + safety stock
- Usually orders are accumulated periodically

