Total Quality Management and Six Sigma

Post Graduate Program 2014-15

Session 4

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Wishing you all a "Total Quality" New Year!

Hope you achieve Six sigma heights

Recap

- Statistical process control
  - Manufacturing and service sector applications
  - Implementation challenges

Agenda

- Magnificent seven SPC tools
- Additional tools for services
- Technical aspects of SPC usage
  - Rational subgrouping
  - Phases of SPC implementation
  - Shewart Control Charts
- Phase 2 implementation
  - CUSUM charts
  - EWMA charts
- Short production runs: DNOM charts, Q charts
- Multiple stream processes
- Economic design of control charts
- Adaptive sampling for better control using SPC

Magnificent seven

- Histogram or stem-and-leaf plot
- Check sheet
- Pareto chart
- Cause-and-effect diagram
- Defect concentration diagram
- Scatter diagram
- Control charts

SPC in services

- Natural measurement systems non-existent
- Observability is low
- Process mapping
- Value stream mapping

Rational subgrouping

- Subgroups or samples should be selected such that
  - Chance of differences between subgroups should be maximized
  - Chance of differences due to assignable causes within a subgroup should be minimized
- Time order is the logical basis
- Snapshot approach v/s random sample approach
- Subgroup based on shifts, machines, operators etc

Phases of Implementation

- Phase 1
  - When SPC is introduced
  - Bringing the process in control
  - Detect high levels of variation
- Phase 2
  - When major assignable causes have been detected and corrected
  - Sustaining the process in control
  - Detect low levels of variation as well
    - Before it is too late!!

Shewart control charts

- Variable control charts
  - X-bar and R charts/s-charts
- Attribute control charts
  - p-charts and c-charts
- Iterative corrective procedure
- Impact of non-normality
- Control chart performance

Control Chart Performance

Average Run Length

- The average run length (ARL) is a very important way of determining the appropriate sample size and sampling frequency.
- Let p = probability that any point exceeds the control limits. Then,

$$ARL = \frac{1}{p} \quad (15\text{-}28)$$

Control Chart Performance

Table 15-6 Average Run Length (ARL) for an $\overline{X}$ Chart with 3-Sigma Control Limits

Magnitude of Process Shift	ARL $n = 1$	ARL $n = 4$
0	370.4	370.4
0.5σ	155.2	43.9
1.0σ	43.9	6.3
1.5σ	15.0	2.0
2.0σ	6.3	1.2
3.0σ	2.0	1.0

Individual measurements

- When there is no basis for rational subgrouping
- Data is available relatively slowly
- Multiple measurements taken on the same unit of the product
- Continuous flow process
- Use moving range MR with the assumption that the same size is 2
- Pros and cons
  - High ARL
  - Normality assumption

The Cumulative-Sum Control Chart

- The cusum chart incorporates all information in the sequence of sample values by plotting the *cumulative sums* of the deviations of the sample values from a target value.
- If μ_0 is the target for the process mean, $\overline{x}_j$ is the average of the jth sample, then the cumulative sum control chart is formed by plotting the quantity

$$C_i = \sum_{j=1}^{i} (\overline{x}_j - \mu_0)$$

The Tabular or Algorithmic Cusum for monitoring the Process Mean

- Let x_i be the ith observation on the process
- If the process is in control then $x_i \sim N(\mu_0, \sigma)$
- Assume σ is known or can be estimated.
- Accumulate derivations from the target μ_0 above the target with one statistic, C^+
- Accumulate derivations from the target μ_0 below the target with another statistic, C^-
- C^+ and C^{--} are one-sided upper and lower cusums, respectively.

The Tabular or Algorithmic Cusum for Monitoring the Process Mean

- The statistics are computed as follows:

The Tabular Cusum

$$C_i^+ = \max\left[0, x_i - (\mu_0 + k) + C_{i-1}^+\right]$$
$$C_i^- = \max\left[0, (\mu_0 - k) - x_i + C_{i-1}^-\right]$$

starting values are $C_0^+ = C_0^- = 0$

K is the **reference value** (or allowance or slack value)

If either statistic exceed a decision interval *H*, the process is considered to be out of control. Often taken as a $H = 5\sigma$

The Tabular or Algorithmic Cusum for Monitoring the Process Mean

Selecting the reference value, *K*

- *K* is often chosen halfway between the target μ_0 and the out-of-control value of the mean μ_1 that we are interested in detecting quickly.
- Shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$, then *K* is

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2}$$

The Tabular or Algorithmic Cusum for Monitoring the Process Mean

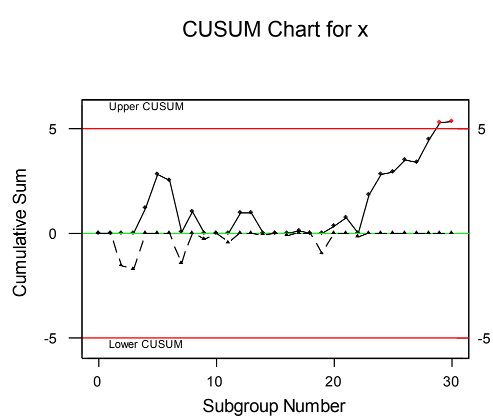
Example 8-1

- $\mu_0 = 10$, n = 1, $\sigma = 1$
- Interested in detecting a shift of $1.0\sigma = 1.0(1.0) = 1.0$
- Out-of-control value of the process mean: $\mu_1 = 10 + 1 = 11$
- $K = ½$ and $H = 5\sigma = 5$ (recommended, discussed in the next section)
- The equations for the statistics are then:

$$C_i^+ = \max\left[0, x_i - 10.5 + C_{i-1}^+\right]$$
$$C_i^- = \max\left[0, 10.5 - x_i + C_{i-1}^-\right]$$

The Tabular or Algorithmic Cusum for Monitoring the Process Mean

Example 8-1



The Tabular or Algorithmic Cusum for Monitoring the Process Mean

Example 8-1

- The cusum control chart indicates the process is out of control.
- The next step is to search for an assignable cause, take corrective action required, and reinitialize the cusum at zero.
- If an adjustment has to be made to the process, may be helpful to estimate the process mean following the shift.

The Standardized Cusums

- It may be of interest to standardize the variable x_i.

$$y_i = \frac{x_i - \mu_0}{\sigma}$$

- The standardized cusums are then

$$C_i^+ = \max\left[0, y_i - k + C_{i-1}^+\right]$$
$$C_i^- = \max\left[0, k - y_i + C_{i-1}^-\right]$$

Improving Cusum Responsiveness for Large Shifts

- Cusum control chart is ***not*** as effective in detecting large shifts in the process mean as the Shewhart chart.
- An alternative is to use a **combined cusum-Shewhart procedure** for on-line control.
- The combined cusum-Shewhart procedure can improve cusum responsiveness to large shifts.

The Fast Initial Response or Headstart Feature

- These procedures were introduced to increase sensitivity of the cusum control chart upon start-up.
- The fast initial response (FIR) or headstart sets the starting values C_0^+, C_0^- equal to some nonzero value, typically H/2.
- Setting the starting values to H/2 is called a 50 percent headstart.

One-Sided Cusums

- There are practical situations where a single one-sided cusum is useful.
- If a shift in only one direction is of interest then a one-sided cusum would be applicable.

The Exponentially Weighted Moving Average Control Chart

The Exponentially Weighted Moving Average Control Chart Monitoring the Process Mean

- The exponentially weighted moving average (EWMA) is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

where $0 < \lambda \leq 1$ is a constant.

$z_0 = \mu_0$ (sometimes $z_0 = \overline{x}$)

8-2.1 The Exponentially Weighted Moving Average Control Chart Monitoring the Process Mean

- The control limits for the EWMA control chart are

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

where *L* is the width of the control limits.

The Exponentially Weighted Moving Average Control Chart Monitoring the Process Mean

- As *i* gets larger, the term [1- (1 - λ)^{2i}] approaches infinity.
- This indicates that after the EWMA control chart has been running for several time periods, the control limits will approach **steady-state** values given by

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

Design of an EWMA Control Chart

- The design parameters of the chart are L and λ.
- The parameters can be chosen to give desired ARL performance.
- In general, $0.05 \leq \lambda \leq 0.25$ works well in practice.
- $L = 3$ works reasonably well (especially with the larger value of λ.
- L between 2.6 and 2.8 is useful when $\lambda \leq 0.1$
- Similar to the cusum, the EWMA performs well against *small shifts* but does not react to large shifts as quickly as the Shewhart chart.
- EWMA is often *superior* to the cusum for larger shifts particularly if $\lambda > 0.1$

Robustness of the EWMA to Non-normality

- As discussed in previously, the **individuals** control chart is sensitive to non-normality.
- A properly designed EWMA is less sensitive to the normality assumption.

DNOM Charts:
Deviation from Nominal

Principles

– Different parts will have different target values

– Calculate the deviation from nominal value

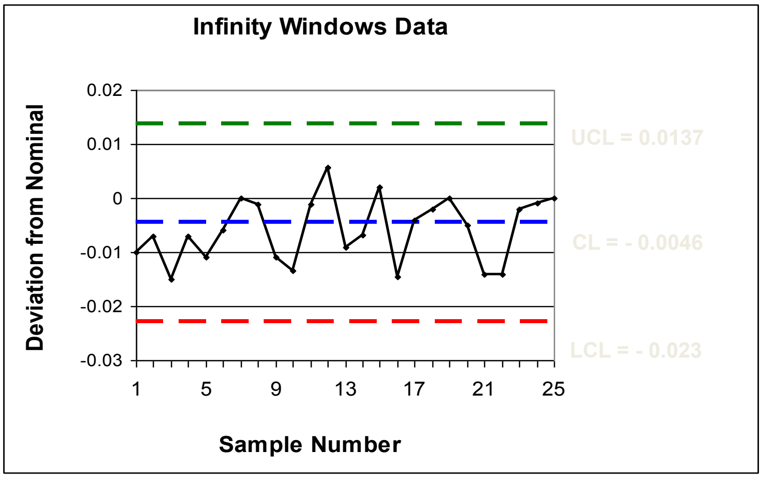
– Plot deviation as the quality characteristic

Infinity Windows Sample Data

- Three part types:
  - Header
  - Right jamb
  - Left jamb
- Nominal length varies from part to part
- Continuous runs; no batches

Part	Date	Time	Nominal Length	Actual Length
Right Jamb	14-Feb	6:51 AM	59.268	59.258
Header	14-Feb	6:54 AM	23	22.993
Header	14-Feb	6:56 AM	35.875	35.86
Right Jamb	14-Feb	7:00 AM	37.518	37.511
Left Jamb	14-Feb	7:08 AM	37.518	37.507
Header	14-Feb	7:12 AM	43.875	43.869
Header	14-Feb	7:14 AM	27.75	27.75
Right Jamb	14-Feb	7:15 AM	37.518	37.5169
Left Jamb	14-Feb	7:18 AM	37.518	37.5071
Header	14-Feb	10:06 AM	39.875	39.8617

DNOM Chart



DNOM Charts

Strengths

- Groups multiple parts and their data sets on a single chart
- Provides a continuous view of the process
- Fairly simple to construct and understand

Shortcomings

- Assumes variation is equal for all parts
- Requires some historical data to calculate control limits
- Does not address quality costs
- Only tracks within-run variation

Standardized Control Charts

Principles

- Multiple part-types flow through a single machine
- Different parts may have different target values
- Control limits and plot points are standardized to allow charting of multiple part-types

Standardized Control Charts

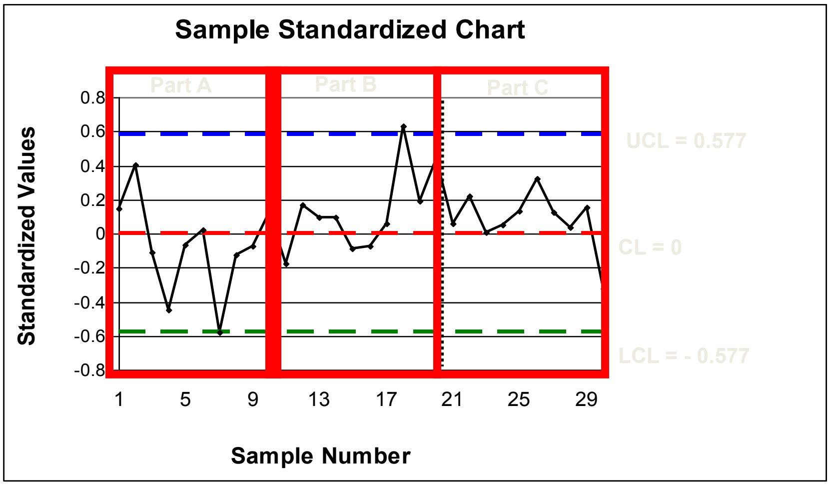
Strengths

- Groups multiple parts and their data sets on a single chart
- Provides a continuous view of the process
- Fairly simple to construct and understand
- Does not assume all parts have equal variation

Shortcomings

- Requires some historical data to calculate control limits
- Does not address quality costs
- Only tracks within-run variation

Sample Standardized Chart



Modified and Acceptance control charts

- When the process is highly capable
  - It might be a good idea to relax the control limits a bit
  - The relaxed control limits could be based on the specification limits.

$$\mu_L = \text{LSL} + Z_\delta \sigma \qquad \text{UCL} = \text{USL} - \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma \qquad = \text{USL} - \left(Z_\gamma + \frac{Z_\beta}{\sqrt{n}}\right)\sigma$$

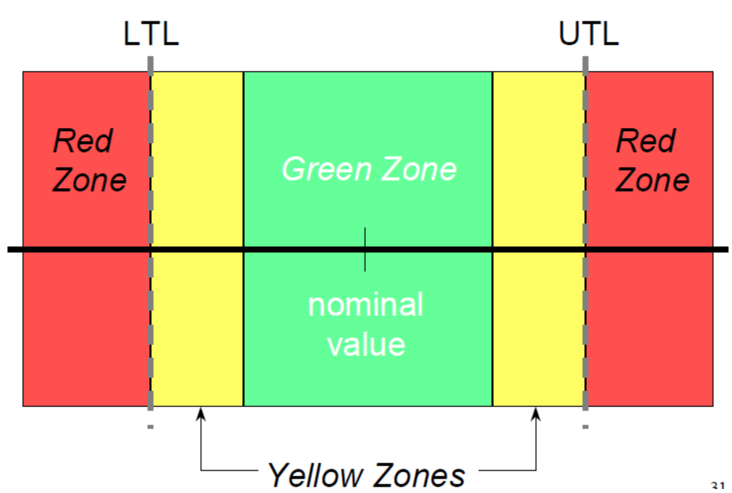
OR

$$\mu_U = \text{USL} - Z_\delta \sigma \qquad \text{LCL} = \text{LSL} + \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma \qquad = \text{LSL} + \left(Z_\gamma + \frac{Z_\beta}{\sqrt{n}}\right)\sigma$$

Multiple stream processes

- Parallel and identical processes
  - Prohibitively large number of control charts
- Assignable causes might impact one or few streams at a time or all streams put together
- Use samples from the processes to formulate the control limits
- Plot max and min values across all the streams
- If the same stream is showing up as max or min value consecutively, then process out of control

Adaptive sampling procedures



Economic design of control charts

- Control chart design has statistical considerations only
- Cost categories to consider
  - Costs of sampling and testing
  - Costs associated with investigating an out of control signal and repair of the assignable cause
  - Costs associated with the production of non-confirming items

THANK YOU